ST CATHERINE'S CONVENT

VERITAS

GRADE 12

AP MATHEMATICS

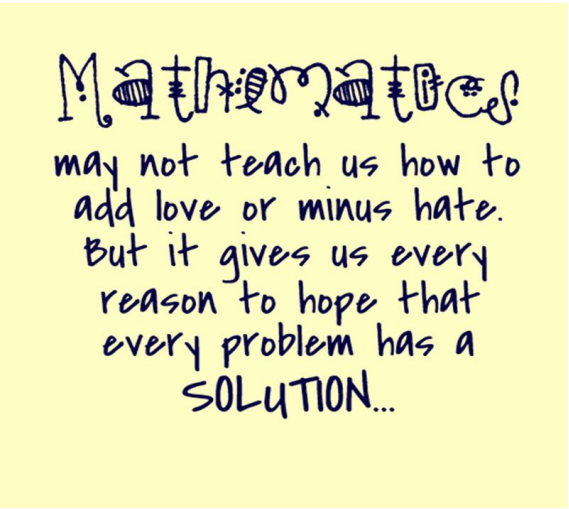
June 2014

EXAMINER: Mrs. A. Rossouw **TOTAL: 200**

TIME: 2 Hours

INSTRUCTIONS:

1. Show all necessary calculations.
2. Round off to **two** decimal places, unless specified otherwise.
3. Non-programmable calculators may be used, unless instructed otherwise.
4. Diagrams are not necessarily drawn to scale.
5. This question paper consists of **5** typed pages and an answer sheet.



Question 1

Prove by induction that

$0{,}1 + 1{,}4 + 2{,}7 + 3{,}10 + \cdots . . + (n-1)(3n-2) = n^2(n-1)$ for all natural numbers , with $n \geq 1$ (14)

Question 2

2.1 Solve for *x* without the use of a calculator:

2.1.1 $|lnx| - 2 = \frac{3}{|lnx|}$ (8)

2.1.2 $2x^2 - 3|x| - 5 = 0$ (8)

2.2 Given: $f(x) = \ln(x+3)$

2.2.1 Find $f^{-1}(x)$ (4)

2.2.2 Sketch the graph of $f(x)$. Indicate all intercepts with the axes as well as all asymptotes clearly. (3)

2.2.3 Find $f \circ f^{-1}(x)$ (4)

2.3 The number of rabbits, K, on an island after t years can be determined by

$K(t) = \frac{3000}{3+7e^{-0{,}05t}}$

2.3.1 Determine the number of rabbits when the experiment started. (3)

2.3.2 Determine the number of rabbits after one year. Give your answer as a whole number. (3)

2.3.3 More than 900 rabits is an epidemic. After how many years will an epidemic occur? (5)

Question 3

3.1 If it is given that $z = x + iy$ and $\bar{z} = x - iy$, find *x* and *y* if $z + 2\bar{z} = \frac{1+i}{2+i}$ (8)

3.2 Factorise $f(x) = x^4 - 4x^3 - 4x - 1$ in C(x) if $2 - \sqrt{5}$ is a zero point of $f(x)$. (8)

3.3 $x = -3 + 4i$ is a root of the equation $-2x^3 + kx^2 - 44x + 25 = 0$. Find the value of k. (8)

Question 4

4.1 Determine c if the function is continuous

$$f(x)=\begin{cases} e^{2x+c} & x \leq 0 \\ x+2 & x>0 \end{cases}$$ (4)

4.2

$$g(x)=\begin{cases} -x^2-6x-5 \ ; & x \leq -3 \\ |2x|-4 \ ; & -3<x\leq 3 \\ -\frac{2}{3}x+4 \ ; & x>3 \end{cases}$$

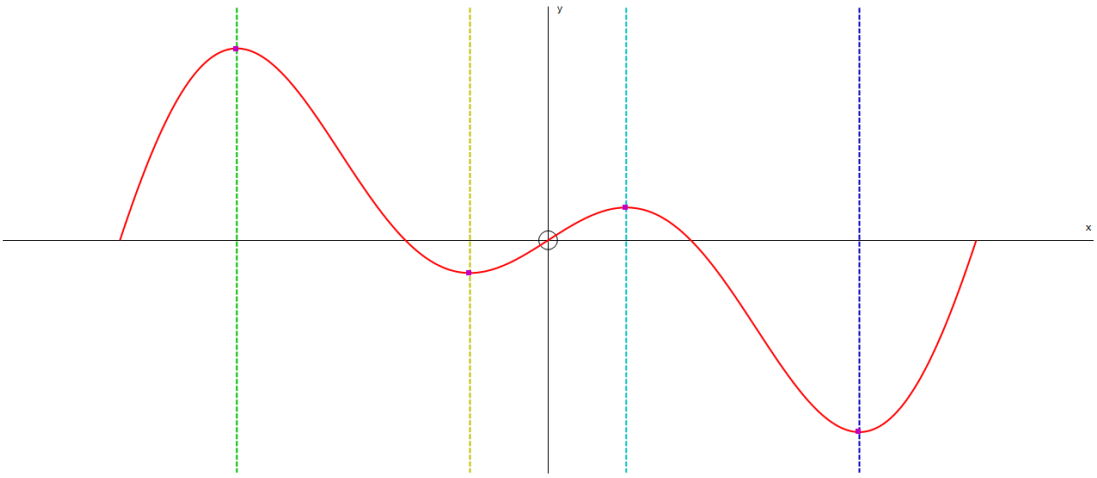
4.1.1 What type of discontinuity exist at $x=-3$? Motivate your answer. (3)

4.1.2 Assume $g(x)$ is continuous at $x=3$. Is $g(x)$ differentiable at $x=3$? Justify your answer fully. (3)

Question 5

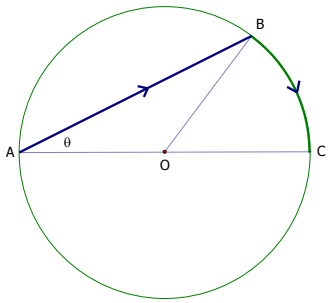
5.1 Determine $f'(x)$ from first principles if $f(x)=\frac{2}{\sqrt{x+5}}$. (6)

5.2 The graph of $f(x)=x\cdot cosx$ is given for $x\in\left[-\frac{3\pi}{2};\frac{3\pi}{2}\right]$



5.2.1 Draw a graph that represents $f'(x)$ on the answer sheet. (6)

5.3 The figure shows a circular lake, centre O, of radius 2 km. A man swims across the lake from A to B at 3km/h and then waks round the edge of the lake from B to C at 4km/h.



5.3.1 Show that $AB = 4cos\theta$ (6)

5.3.2 Find the length of CB in terms of θ (2)

5.3.3 Show that the total time taken in hours , T, is $T = \frac{4}{3}cos\theta + \theta$ (4)

5.3.4 Find the value of θ for which T is a maximum. (6)

5.4 Determine the gradient of the tangent to $x^2e^y + y^2e^x = 2e$ at $(1; 1)$ (5)

Question 6

Given $f(x) = 2x - secx$

6.1 Prove that the given function has a zero point between $x = 0$ and $x = \frac{3}{4}$ (3)

6.2 Use $x_0 = 0{,}895$ and Newton's method to determine the zero point correct to 5 decimal places. (6)

6.3 Give the formula that will be used to determine the x-value of the turning point. (3)

Question 7

7.1 Evaluate:

7.1.1 $\int \left(x^5 + \frac{2}{x^2} - \sqrt{x}\right) dx$ (4)

7.1.2 $\int cos2x \cdot cos4x \, dx$ (6)

7.1.3 $\int_0^{\frac{\pi}{2}} x \cdot cos3x \ dx$

(10)

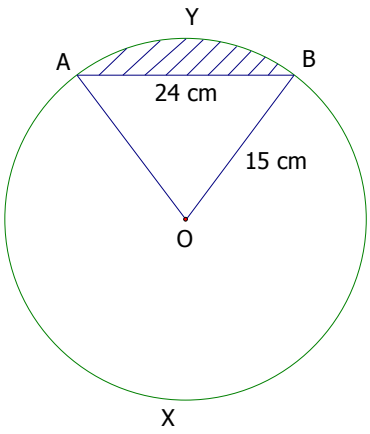
7.1.4 $\int \frac{6x-5}{\sqrt{-3x^2+5x-6}} dx$ (6)

7.2 Determine the volume if $1 + cosx$ rotate around the $x - axis$ from $x\varepsilon \left[0; \frac{\pi}{2}\right]$.

Give your answer in terms of π (6)

Question 8

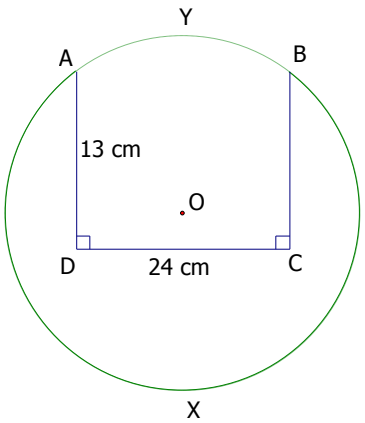
8.1 The figure shows a cross-section of a log with centre O and radius 15 cm. Chord AB is 24 cm in length.



8.1.1 Show that $A\hat{O}B = 1{,}85\ radians$ (4)

8.1. 2 Find the area of the shaded region. (6)

8.2 The following is a cross-section of the same log with section ADCBYA removed throughout the length of the log.



Given that $AD = BC = 13\ cm$ and that $ABCD$ is a rectangle, find the area of the new cross-section , $AXBCDA$ (6)

Question 9

Given : $f(x) = \frac{x^2-2x+1}{x+1}$

9.1 Determine the asymptotes of the rational function. (5)

9.2 Determine the coordinates of the stationary points of the graph. (8)

9.3 Determine the intercepts with the axes. (2)

9.4 Sketch the graph. Indicate all asymptotes, stationary points and intercepts with axes cleary. (4)

NAME_____

ANSWER SHEET

QUESTION 5.2